Finite Element Analysis in a Rotating Thermoelastic Half-Space with Diffusion

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A thermal shock problem of rotating generalized thermoelastic half-space with diffusion is considered. The governing equations of the model are reduced in xy-plane in terms of non-dimensional quantities. The corresponding finite element equations are obtained, the solutions of which are subjected to required initial and boundary conditions. Using finite element method, the distributions of displacement components, stress components, temperature, chemical potential and mass concentration are computed for copper material as a particular material of the model. These distributions are shown graphically against the distance from boundary to observe the effect of rotation parameter.

Keywords: Lord and Shulman Theory, Thermodiffusion, Finite Element Method.

1. INTRODUCTION

The classical theory of thermoelasticity, the foundations of which were laid in the nineteenth century by Duhamel, Neumann and Lord Kelvin, is based on Fourier's law of heat conduction.[1] When combined with the law of conservation of energy, Fourier's law gives rise to the well-known diffusion, or heat, equation as the partial differential equation governing heat transport. Unfortunately, the parabolic nature of this equation implies that a thermal disturbance at any point in a material body, in particular, a thermoelastic solid, will be felt instantly, but unequally, at all other points of the body; in other words, Fourier's law predicts that thermal signals propagate with infinite speed, and thus conflicts with the requirements of causality. To correct this unrealistic feature, which has come to be known as the 'paradox of heat conduction,' and its resulting impact on thermoelasticity theory, various modifications to classical theory of thermoelasticity have been proposed. For example, Lord and Shulman[2] developed the theory of generalized thermoelasticity with one relaxation time for the special case of an isotropic body. Green and Lindsay[3] developed the theory of thermoelasticity after taking two relaxation times. The above two theories allow finite speed of propagation of waves. Recently, Ignaczak and Ostoja-Starzewski[4] presented the analysis of above two theories in their recent book on "Thermoelasticity with

Finite Wave Speeds." Chandrasekharaiah[5] referred to this wavelike thermal disturbance as "second sound." The representative theories in the range of generalized thermoelasticity are reviewed by Hetnarski and Ignaczak.[6]

Thermodiffusion in an elastic solid is due to the field of temperature, mass diffusion and that of strain. The diffusion phenomenon is of great concern due to its many geophysical and industrial applications. The concept of thermodiffusion is helpful to oil companies for more efficient extraction of oil from oil deposits. Nowacki[7, 8] developed coupled thermodiffusion model in solids and studied some dynamical problems on thermodiffusion in solids. Dudziak and Kowalski[9] discussed the theory of thermodiffusion for solid. Olesiak and Pyryev[10] studied a coupled quasi-stationary problem of thermodiffusion for elastic cylinder. They discussed the influences of cross effects arising of the coupling of the fields of temperature, mass diffusion and strain. Due to these cross effects the thermal excitation results in an additional mass concentration and the mass concentration generates the additional field of temperature. Following Lord and Shulman[2] theory, Sherief et al.[11] developed a theory of generalized thermoelastic diffusion, which also allow the finite speeds of propagation of waves. Singh[12, 13] studied the wave propagation in thermoelastic solid with diffusion in context of both Lord-Shulman and Green-Lindsay theories. Various other problems on the theory of generalized thermoelastic diffusion were also studied during last few years.[14–17]

RESEARCH ARTICLE

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The exact solution of the governing equations for a linear/nonlinear classical and non-classical thermoelastic theories exists for very special and simple initial and boundary problem only. To solve analytically the general boundary value problems in thermoelasticity with various additional parameters is tedious job, therefore numerical solution techniques like finite element method (FEM) are applied to solve such problems. The method of weighted residuals helps us in formulation of the finite element equations for a selected model, which gives a best approximated solutions to linear/nonlinear ordinary and partial differential equations. There are three main steps in applying the FEM for a model. The first step is to assume the general behavior of the unknown field variables in such a way that these satisfy the given differential equations. These approximating field variables are then substituted into the differential equations and boundary conditions which results in some errors, called the residual. This residual has to vanish in an average sense over the solution domain. The second step is the time integration. The time derivatives of the unknown variables have to be determined by former results. The equations resulting from the first and the second step are solved in last step by solving an algorithm of the finite element program.[18] The advantages of the finite element method, as compared to other numerical approaches, are numerous. The method is completely general with respect to geometry and material properties. Complex bodies composed of many different anisotropic materials are easily represented. Temperature or heat flux boundary conditions may be specified at any point within the finite element system. Mathematically, it can be shown that the method converges to the exact solution as the number of elements is increased; therefore, any desired degree of accuracy may be obtained. Finite element method has been used for numerical solution of various thermoelastic problems.[19–23] Recently, Refs. [24–26] variants problems in waves are studied. Other forms are described for example in the Refs. [27–29].

The effects of rotation, magnetic and temperature variation on speed or frequency of wave provide the basis for the development of many acoustic sensors.[30] Particularly, frequency shifts due to rotation have been used to make gyroscopes.[31, 32] Schoenberg and Censor[33] studied the effect of rotation on plane wave propagation in an isotropic medium. They showed the propagation of three plane waves in a rotating isotropic medium. They found that the longitudinal or transverse wave can exist only if the direction of propagation and axis of rotation are either parallel or perpendicular.

In the present paper, we have considered a thermal shock problem in a half-space of a rotating elastic solid with thermodiffusion. Numerical results for the displacement, temperature and concentration distributions are obtained and are displayed graphically to observe the effect of rotation parameter.

2. BASIC EQUATIONS

Following Sherief[11] and Schoenberg and Censor,[33] the basic governing equations of generalized thermoelasticity with diffusion and rotation are

(i) the equations of motion

$$\mu u_{i,jj} + (\lambda + \mu) u_{j,ij} - \gamma \theta_{,i} - \beta C_{,i}$$
$$= \rho[\ddot{u}_i + \{\Omega \times (\Omega \times u)\}_i + (2\Omega \times \dot{u})_i] \quad (1)$$

(ii) the equation of heat conduction

$$K\theta_{,ii} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) (\rho c_e \theta + a T_0 C + \gamma T_0 e) \quad (2)$$

(iii) the equation of mass diffusion

$$DbC_{,ii} - Da\theta_{,ii} - D\beta e_{,ii} = \left(\frac{\partial}{\partial t} + \tau_1 \frac{\partial^2}{\partial t^2} \right)^2 C \quad (3)$$

(iv) the constitutive equations

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}(\lambda e_{kk} - \gamma\theta - \beta C)$$
$$P = -\beta e_{kk} + bC - a\theta \quad (4)$$
$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

where list of symbols is given in the nomenclature. The superposed dots denote the time partial derivatives. The subscripts followed by comma denote the space partial differentiations.

3. FORMULATION OF THE PROBLEM

We consider a homogeneous, isotropic, generalized thermodiffusive elastic half space initially at uniform temperature T_0. We use a fixed Cartesian coordinate system (x, y, z) with origin on the surface $x = 0$, which is stress free and with y-axis directed vertically into the medium. The region $x > 0$ is occupied by the elastic solid with generalized thermodiffusion. We restrict our analysis parallel to xy-plane. We assume that all quantities are functions of the coordinates x, y and time t and independent of coordinate z. So the components of displacement vector, temperature and concentration can be taken in the following form

$$u = u_x = u(x, y, t), \quad v = u_y = v(x, y, t),$$
$$w = u_z = 0, \quad T = T(x, y, t), \quad C = C(x, y, t) \quad (5)$$

From Eqs. (4) and (5), we can obtain the constitutive equations

$$e_{xx} = \frac{\partial u}{\partial x}, \quad e_{yy} = \frac{\partial v}{\partial x}, \quad e_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \quad (6)$$

$$e_{zz} = 0, \quad e_{xz} = 0, \quad e_{yz} = 0$$

$$\sigma_{xx} = 2\mu \frac{\partial u}{\partial x} + \lambda e - \gamma\theta - \beta C \quad (7)$$

$$\sigma_{yy} = 2\mu \frac{\partial v}{\partial y} + \lambda e - \gamma\theta - \beta C \quad (8)$$

$$\sigma_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \tag{9}$$

$$P = -\beta e + bC - a\theta \tag{10}$$

$$e = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \tag{11}$$

Equations (1)–(3) thus reduce to

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2} + (\lambda+\mu)\frac{\partial^2 v}{\partial x\partial y} + \mu\frac{\partial^2 u}{\partial y^2}$$
$$= \gamma\frac{\partial \theta}{\partial x} + \beta\frac{\partial C}{\partial x} + \rho\frac{\partial^2 u}{\partial t^2} - \Omega^2 u - 2\Omega\frac{\partial v}{\partial t} \tag{12}$$

$$(\lambda+2\mu)\frac{\partial^2 v}{\partial y^2} + (\lambda+\mu)\frac{\partial^2 u}{\partial x\partial y} + \mu\frac{\partial^2 v}{\partial x^2}$$
$$= \gamma\frac{\partial \theta}{\partial y} + \beta\frac{\partial C}{\partial y} + \rho\frac{\partial^2 v}{\partial t^2} - \Omega^2 v + 2\Omega\frac{\partial u}{\partial t} \tag{13}$$

$$k\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) = \left(\frac{\partial}{\partial t} + t_0\frac{\partial^2}{\partial t^2}\right)\left(\rho c_e \theta + aT_0 C\right.$$
$$\left. + T_0\gamma\frac{\partial u}{\partial x} + T_0\gamma\frac{\partial v}{\partial y}\right) \tag{14}$$

$$Db\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) - Da\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right)$$
$$- D\beta\left(\frac{\partial^3 u}{\partial x^3} + \frac{\partial^3 u}{\partial x\partial y^2} + \frac{\partial^3 v}{\partial y^3} + \frac{\partial^3 v}{\partial y\partial x^2}\right)$$
$$= \left(\frac{\partial}{\partial t} + t_1\frac{\partial^2}{\partial t^2}\right)C \tag{15}$$

Introducing the following non-dimensional variables

$$(x', y', u', v') = \frac{\eta}{c_1}(x, y, u, v)$$

$$(t', t_0', t_1') = \eta(t, t_0, t_1),$$

$$\theta' = \frac{\gamma\theta}{\rho c_1^2}, \quad C' = \frac{\beta C}{\rho c_1^2}, \quad \Omega' = \frac{\Omega}{\eta}, \tag{16}$$

$$(\sigma_{xx}', \sigma_{xy}', \sigma_{yy}') = \frac{1}{\rho c_1^2}(\sigma_{xx}, \sigma_{xy}, \sigma_{yy}),$$

$$P' = \frac{P}{\beta}, \quad c_1^2 = \frac{\lambda+2\mu}{\rho}, \quad \eta = \frac{\rho c_e c_1^2}{k}$$

In terms of the non-dimensional quantities defined in Eq. (16), the above governing equations reduce to (dropping the dashed for convenience)

$$\sigma_{xx} = \frac{\partial u}{\partial x} + a_1\frac{\partial v}{\partial y} - \theta - C \tag{17}$$

$$\sigma_{yy} = \frac{\partial v}{\partial y} + a_1\frac{\partial u}{\partial x} - \theta - C \tag{18}$$

$$\sigma_{xy} = a_2\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \tag{19}$$

$$P = -\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} + \xi_1 C - \xi_2\theta \tag{20}$$

$$\frac{\partial^2 u}{\partial x^2} + (a_1+a_2)\frac{\partial^2 v}{\partial x\partial y} + a_2\frac{\partial^2 u}{\partial y^2}$$
$$= \frac{\partial \theta}{\partial x} + \frac{\partial C}{\partial x} + \frac{\partial^2 u}{\partial t^2} - \Omega^2 u - 2\Omega\frac{\partial v}{\partial t} \tag{21}$$

$$\frac{\partial^2 v}{\partial y^2} + (a_1+a_2)\frac{\partial^2 u}{\partial x\partial y} + a_2\frac{\partial^2 v}{\partial x^2}$$
$$= \frac{\partial \theta}{\partial y} + \frac{\partial C}{\partial y} + \frac{\partial^2 v}{\partial t^2} - \Omega^2 v + 2\Omega\frac{\partial u}{\partial t} \tag{22}$$

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = \left(\frac{\partial}{\partial t} + t_0\frac{\partial^2}{\partial t^2}\right)$$
$$\times \left(\theta + \varepsilon_c C + \varepsilon_e\frac{\partial u}{\partial x} + \varepsilon_e\frac{\partial v}{\partial y}\right) \tag{23}$$

$$\xi_1\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) - \xi_2\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) - \left(\frac{\partial^3 u}{\partial x^3} + \frac{\partial^3 v}{\partial x^2\partial y}\right.$$
$$\left. + \frac{\partial^3 u}{\partial y^2\partial x} + \frac{\partial^3 v}{\partial y^3}\right) = \xi_3\left(\frac{\partial}{\partial t} + t_1\frac{\partial^2}{\partial t^2}\right)C \tag{24}$$

where, $\quad a_1 = \frac{\lambda}{\lambda+2\mu}, \quad a_2 = \frac{\mu}{\lambda+2\mu}, \quad \varepsilon_c = \frac{aT_0\gamma}{\rho c_e\beta}$

$$\varepsilon_e = \frac{T_0\gamma^2}{\rho^2 c_e c_1^2}, \quad \xi_1 = \frac{b\rho c_1^2}{\beta^2}, \quad \xi_2 = \frac{a\rho c_1^2}{\gamma\beta}, \quad \xi_3 = \frac{\rho c_1^4}{D\eta\beta^2}$$

The above equations are solved subjected to the initial conditions

$$u = v = \theta = C = 0, \quad t = 0$$
$$\dot{u} = \dot{v} = \dot{\theta} = \dot{C} = 0, \quad t = 0 \tag{25}$$

The boundary conditions for the problem may be taken as

$$\theta(0, y, t) = \theta_o H(t)H(2l - |y|), \quad \sigma_{xx}(0, y, t) = 0$$
$$\sigma_{xy}(0, y, t) = 0, \quad P(0, y, t) = 0 \tag{26}$$

where, H is the Heaviside unit step.

4. FINITE ELEMENT METHOD

In this section, the governing equations of generalized thermoelastic diffusion with relaxation times (Lord and Shulman theory) are summarized, followed by the corresponding finite element equations. In the finite element method, the displacement components u, v, temperature θ and concentration C are related to the corresponding nodal values by

$$u = \sum_{i=1}^{m} N_i u_i(t), \quad v = \sum_{i=1}^{m} N_i v_i(t),$$
$$\theta = \sum_{i=1}^{m} N_i \theta_i(t), \quad C = \sum_{i=1}^{m} N_i C_i(t) \tag{27}$$

where m denotes the number of nodes per element, and N_i are the shape functions. The eight-node isoparametric,

quadrilateral element is used for displacement components, temperature and concentration calculations. The weighting functions and the shape functions coincide. Thus,

$$\delta u = \sum_{i=1}^{m} N_i \delta u_i, \quad \delta v = \sum_{i=1}^{m} N_i \delta v_i$$
$$\delta \theta = \sum_{i=1}^{m} N_i \delta \theta_i, \quad \delta C = \sum_{i=1}^{m} N_i \delta C_i \tag{28}$$

It should be noted that appropriate boundary conditions associated with the governing Eqs. (21)–(24) must be adopted in order to properly formulate a problem. Boundary conditions are either essential (or geometric) or natural (or traction) types. Essential conditions are prescribed displacements u, v, temperature θ and concentration C while, the natural boundary conditions are prescribed tractions, heat flux and mass flux which are expressed as

$$\sigma_{xx} n_x + \sigma_{xy} n_y = \bar{\tau}_x, \quad \sigma_{xy} n_x + \sigma_{yy} n_y = \bar{\tau}_y$$
$$q_x n_x + q_y n_y = \bar{q}, \quad \eta_x n_x + \eta_y n_y = \bar{\eta} \tag{29}$$

where n_x and n_y are direction cosines of the outward unit normal vector at the boundary, $\bar{\tau}_x$, $\bar{\tau}_y$ are the given tractions values, $\bar{q}$ is the given surface heat flux and $\bar{\eta}$ is the given surface mass flux.

In the absence of body force, the governing equations are multiplied by weighting functions and then are integrated over the spatial domain Ψ with the boundary Γ. Applying integration by parts and making use of the divergence theorem reduce the order of the spatial derivatives and allows for the application of the boundary conditions. Thus, the finite element equations corresponding to Eqs. (21)–(24) can be obtained as

$$\int_{\Psi} \left\{ \begin{array}{c} \dfrac{\partial \delta u}{\partial x} \sigma_{xx} + \dfrac{\partial \delta u}{\partial y} \sigma_{xy} \\[2mm] \dfrac{\partial \delta v}{\partial x} \sigma_{xy} + \dfrac{\partial \delta v}{\partial y} \sigma_{yy} \\[2mm] \left(\dfrac{\partial \delta \theta}{\partial x} \dfrac{\partial \theta}{\partial x} + \dfrac{\partial \delta \theta}{\partial y} \dfrac{\partial^2 \theta}{\partial t \partial y} \right) \\[2mm] \left(\dfrac{\partial \delta C}{\partial x} \dfrac{\partial P}{\partial x} + \dfrac{\partial \delta C}{\partial y} \dfrac{\partial P}{\partial y} \right) \end{array} \right\} d\Psi$$

$$+ \int_{\Psi} \left\{ \begin{array}{c} \delta u \left(\dfrac{\partial^2 u}{\partial t^2} - \Omega^2 u - 2\Omega \dfrac{\partial v}{\partial t} \right) \\[2mm] \delta v \left(\dfrac{\partial^2 v}{\partial t^2} - \Omega^2 v + 2\Omega \dfrac{\partial u}{\partial t} \right) \\[2mm] \delta \theta \left(\dfrac{\partial}{\partial t} + t_0 \dfrac{\partial^2}{\partial t^2} \right) (\theta + \varepsilon_c C \\[2mm] \qquad\qquad + \varepsilon_e \dfrac{\partial u}{\partial x} + \varepsilon_e \dfrac{\partial v}{\partial y}) \\[2mm] \xi_3 \left(\dfrac{\partial}{\partial t} + t_1 \dfrac{\partial^2}{\partial t^2} \right) C \end{array} \right\} d\Psi$$

$$= \int_{\Gamma} \left\{ \begin{array}{c} \delta u \bar{\tau}_x \\[1mm] \delta v \bar{\tau}_y \\[1mm] \delta \theta \bar{q} \\[1mm] \delta C \bar{\eta} \end{array} \right\} d\Gamma \tag{30}$$

Substituting the constitutive relations (17)–(20) and Eqs. (28) and (29) into Eq. (30) leads

$$\sum_{e=1}^{me} \left(\begin{bmatrix} M_{11}^e & 0 & 0 & 0 \\ 0 & M_{22}^e & 0 & 0 \\ M_{31}^e & M_{32}^e & M_{33}^e & M_{34}^e \\ 0 & 0 & 0 & M_{44}^e \end{bmatrix} \left\{ \begin{array}{c} \ddot{u}^e \\ \ddot{v}^e \\ \ddot{\theta}^e \\ \ddot{C}^e \end{array} \right\} \right.$$

$$+ \begin{bmatrix} 0 & C_{12}^e & 0 & 0 \\ C_{21}^e & 0 & 0 & 0 \\ C_{31}^e & C_{32}^e & C_{33}^e & C_{34}^e \\ 0 & 0 & 0 & C_{44}^e \end{bmatrix} \left\{ \begin{array}{c} \dot{u}^e \\ \dot{v}^e \\ \dot{\theta}^e \\ \dot{C}^e \end{array} \right\}$$

$$+ \begin{bmatrix} K_{11}^e & K_{12}^e & K_{13}^e & K_{14}^e \\ K_{21}^e & K_{22}^e & K_{23}^e & K_{24}^e \\ 0 & 0 & K_{33}^e & 0 \\ K_{41}^e & K_{42}^e & K_{43}^e & K_{44}^e \end{bmatrix} \left\{ \begin{array}{c} u^e \\ v^e \\ \theta^e \\ C^e \end{array} \right\}$$

$$\left. = \left\{ \begin{array}{c} F_1^e \\ F_2^e \\ F_3^e \\ F_4^e \end{array} \right\} \right) \tag{31}$$

where *me* is the total number of elements. The coefficients in Eq. (31) are given below.

$$M_{11}^e = \int_{\Psi} [N]^T [N] \, d\Psi, \quad M_{22}^e = \int_{\Psi} [N]^T [N] \, d\Psi$$

$$M_{31}^e = \int_{\Psi} t_0 [N]^T [N] \, d\Psi, \quad M_{32}^e = \int_{\Psi} t_0 \varepsilon_c [N]^T [N] \, d\Psi$$

$$M_{33}^e = \int_{\Psi} t_0 \varepsilon_e [N]^T \left[\frac{\partial N}{\partial x} \right] d\Psi$$

$$M_{34}^e = \int_{\Psi} t_0 \varepsilon_e [N]^T \left[\frac{\partial N}{\partial y} \right] d\Psi, \quad C_{31}^e = \int_{\Psi} [N]^T [N] \, d\Psi$$

$$C_{32}^e = \int_{\Psi} \varepsilon_c [N]^T [N] \, d\Psi, \quad C_{33}^e = \int_{\Psi} \varepsilon_e [N]^T \left[\frac{\partial N}{\partial x} \right] d\Psi$$

$$C_{34}^e = \int_{\Psi} \varepsilon_e [N]^T \left[\frac{\partial N}{\partial y} \right] d\Psi, \quad C_{44}^e = \int_{\Psi} \xi_3 [N]^T [N] \, d\Psi$$

$$M_{44}^e = \int_{\Psi} t_1 \xi_3 [N]^T [N] \, d\Psi$$

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$$K_{11}^e = \int_\Psi \left(\left[\frac{\partial N}{\partial x} \right]^T \left[\frac{\partial N}{\partial x} \right] + a_2 \left[\frac{\partial N}{\partial y} \right]^T \left[\frac{\partial N}{\partial y} \right] \right.$$
$$\left. - \Omega^2 [N]^T [N] \right) d\Psi$$

$$K_{13}^e = \int_\Psi - \left[\frac{\partial N}{\partial x} \right]^T [N] \, d\Psi$$

$$K_{12}^e = \int_\Psi (a_1 + a_2) \left[\frac{\partial N}{\partial x} \right]^T \left[\frac{\partial N}{\partial y} \right] d\Psi$$

$$K_{14}^e = \int_\Psi - \left[\frac{\partial N}{\partial x} \right]^T [N] \, d\Psi$$

$$K_{21}^e = \int_\Psi (a_1 + a_2) \left[\frac{\partial N}{\partial y} \right]^T \left[\frac{\partial N}{\partial x} \right] d\Psi$$

$$K_{23}^e = \int_\Psi - \left[\frac{\partial N}{\partial y} \right]^T [N] \, d\Psi$$

$$K_{22}^e = \int_\Psi \left(\left[\frac{\partial N}{\partial y} \right]^T \left[\frac{\partial N}{\partial y} \right] + a_2 \left[\frac{\partial N}{\partial x} \right]^T \left[\frac{\partial N}{\partial x} \right] \right.$$
$$\left. - \Omega^2 [N]^T [N] \right) d\Psi$$

$$K_{24}^e = \int_\Psi - \left[\frac{\partial N}{\partial y} \right]^T [N] \, d\Psi$$

$$K_{33}^e = \int_\Psi \left(\left[\frac{\partial N}{\partial x} \right]^T \left[\frac{\partial N}{\partial x} \right] + \left[\frac{\partial N}{\partial y} \right]^T \left[\frac{\partial N}{\partial y} \right] \right) d\Psi$$

$$K_{41}^e = - \int_\Psi \left(\left[\frac{\partial N}{\partial x} \right]^T \left[\frac{\partial^2 N}{\partial x^2} \right] + \left[\frac{\partial N}{\partial y} \right]^T \left[\frac{\partial^2 N}{\partial x \partial y} \right] \right) d\Psi$$

$$K_{42}^e = - \int_\Psi \left(\left[\frac{\partial N}{\partial x} \right]^T \left[\frac{\partial^2 N}{\partial x \partial y} \right] + \left[\frac{\partial N}{\partial y} \right]^T \left[\frac{\partial^2 N}{\partial y^2} \right] \right) d\Psi$$

$$K_{43}^e = -\xi_2 \int_\Psi \left(\left[\frac{\partial N}{\partial x} \right]^T \left[\frac{\partial N}{\partial x} \right] + \left[\frac{\partial N}{\partial y} \right]^T \left[\frac{\partial N}{\partial y} \right] \right) d\Psi$$

$$K_{44}^e = \xi_1 \int_\Psi \left(\left[\frac{\partial N}{\partial x} \right]^T \left[\frac{\partial N}{\partial x} \right] + \left[\frac{\partial N}{\partial y} \right]^T \left[\frac{\partial N}{\partial y} \right] \right) d\Psi$$

$$F_1^e = \int_\Gamma [N]^T \bar{\tau}_x \, d\Gamma, \quad F_2^e = \int_\Gamma [N]^T \bar{\tau}_y \, d\Gamma$$

$$F_3^e = \int_\Gamma [N]^T \bar{q} \, d\Gamma, \quad F_4^e = \int_\Gamma [N]^T \bar{\eta} \, d\Gamma$$

Symbolically, the discredited equations of Eq. (31) can be written as

$$M \ddot{d} + S \dot{d} + K d = F^{\text{ext}} \qquad (32)$$

where M, S, K and F^{ext} represent the mass, damping, stiffness matrices and external force vectors, respectively; $d = [u \ v \ \theta \ C]^T$. On the other hand, the time derivatives of the unknown variables have to be determined by Newmark time integration method (see Wriggers[34]).
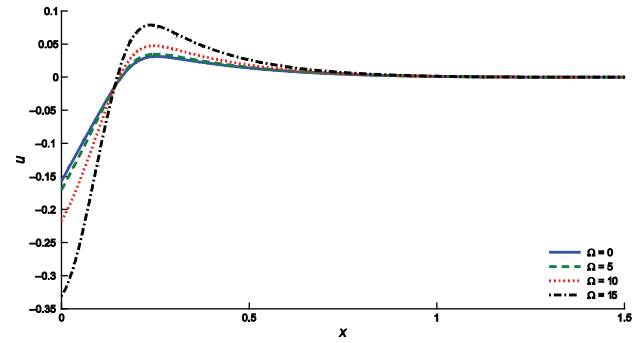

Fig. 1. Tangential displacement distribution.

5. NUMERICAL RESULTS AND DISCUSSION

Following Sherief and Saleh,[35] the physical constants of copper material are chosen for the purpose of numerical calculation.

$$\rho = 8954 \text{ kg m}^{-3}, \quad \lambda = 7.76 \times 10^{10} \text{ kg m}^{-1} \text{ s}^{-2}$$
$$\mu = 3.86 \times 10^{10} \text{ kg m}^{-1} \text{ s}^{-2}, \quad l = 0.5$$
$$c_e = 383.1 \text{ J kg}^{-1} \text{ k}^{-1}, \quad \alpha_t = 1.78 \times 10^{-5} \text{ k}^{-1}$$
$$\alpha_c = 1.98 \times 10^{-4} \text{ m}^3 \text{ kg}^{-1}, \quad K = 386 \text{ w m}^{-1} \text{ k}^{-1}$$
$$T_o = 293 \text{ k}, \quad D = 0.85 \times 10^{-8} \text{ kg s m}^{-3}$$
$$a = 1.2 \times 10^4 \text{ m}^2 \text{ s}^{-2} \text{ k}^{-1}, \quad b = 0.9 \times 10^6 \text{ m}^5 \text{ kg}^{-1} \text{ s}^{-2}$$

The distribution of tangential component u of the displacement field is shown against the distance from boundary plane $x = 0$ in Figure 1. For $\Omega = 0$, 5, 10, 15, the tangential displacement component u is minimum at $x = 0$ and it increases sharply to its maximum values with the increase in distance from boundary and then decreases slowly. At lower distances, the effect of rotation is observed significantly, whereas at greater distances, the effect of rotation vanishes.

In Figure 2, the distribution of normal component v of the displacement field is displayed against the distance from boundary plane $x = 0$. For $\Omega = 0$, 5, 10, 15, the normal displacement component v is maximum at $x = 0$ and it decreases sharply to its minimum values with the
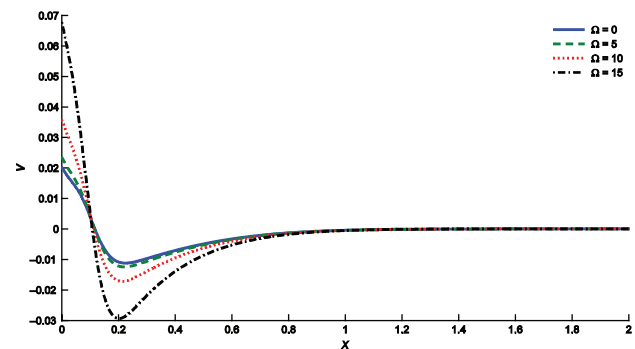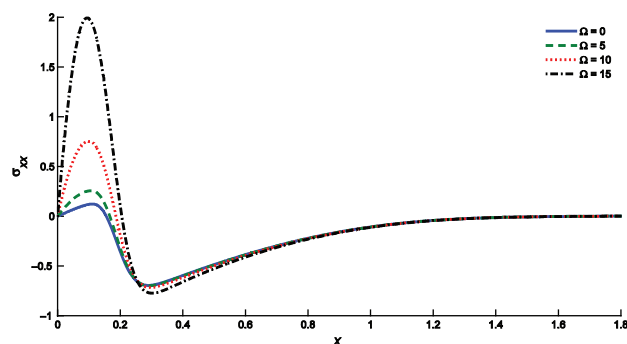

Fig. 2. Normal displacement distribution.

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Fig. 3. Normal stress $\_\xi\xi$ distribution.

Fig. 6. Temperature distribution.

increase in distance from boundary plane $x = 0$ and then increases slowly. For lower distances, the effect of rotation is observed significantly and the effect of rotation vanishes at greater distances.

The distribution of normal stress component σ_{xx} is shown in Figure 3 against the distance from boundary plane $x = 0$. For $\Omega = 0, 5, 10, 15$, the stress component σ_{xx} increases sharply to respective maximum values and then decreases sharply to respective minimum values. Thereafter, it increases slowly with the distance. For distances near boundary plane $x = 0$, the effect of rotation on stress σ_{xx} is observed significantly and the effect decreases with distance and it vanishes at greater distances.

The distribution of tangential stress component σ_{xy} is displayed in Figure 4 against the distance from boundary plane. For $\Omega = 0, 5, 10, 15$, the stress component σ_{xy}
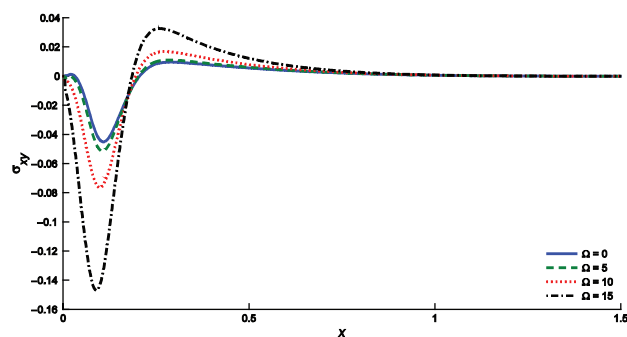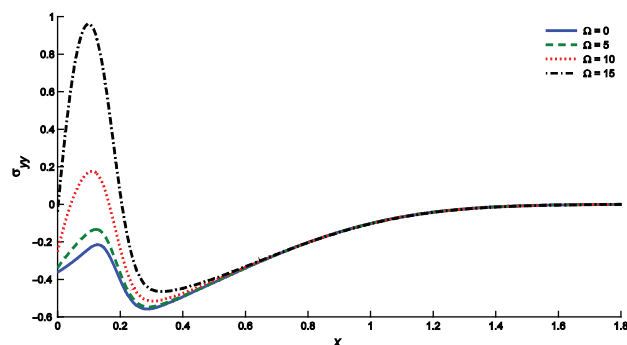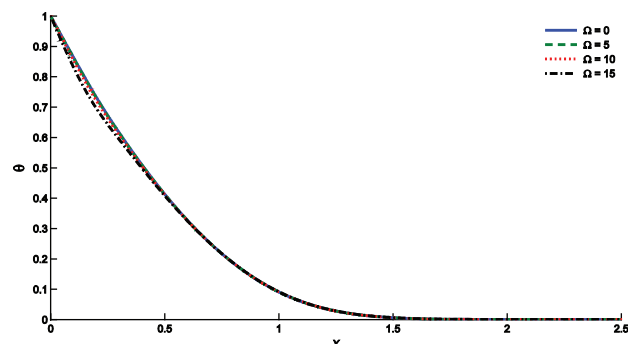

Fig. 4. Tangential stress $\_\xi\psi$ distribution.

Fig. 5. Normal stress $\_\psi\psi$ distribution.

decreases sharply to respective minimum values and then increases sharply to respective maximum values. Thereafter, it decreases slowly with the distance. For distances near boundary plane $x = 0$, the effect of rotation on stress σ_{xy} is observed significantly and the effect decreases with distance and it vanishes at greater distances.

In Figure 5, the distribution stress component σ_{yy} is shown against the distance from boundary plane $x = 0$. For $\Omega = 0, 5, 10, 15$, the stress component σ_{yy} has similar variations as shown for σ_{xx}. The effect of rotation on stress σ_{yy} is observed significantly for distances near boundary plane $x = 0$ and the effect vanishes at greater distances.

Figure 6 displays the temperature distribution θ against the distance from boundary plane $x = 0$. For $\Omega = 0, 5, 10, 15$, the temperature θ decreases with the increase in distance from boundary. The effect of rotation on temperature θ is observed for distances near to the boundary plane $x = 0$ for larger values of rotation parameter Ω.

In Figure 7, the chemical potential distribution P is shown against the distance from boundary plane $x = 0$. For $\Omega = 0, 5, 10, 15$, the chemical potential P first decreases and then increases slowly with the distance from boundary plane. The effect of rotation on chemical potential P is observed for distances near to the boundary plane $x = 0$ for larger values of rotation parameter Ω.

The mass concentration distribution C is shown in Figure 8 against the distance from boundary plane $x = 0$. For $\Omega = 0, 5, 10, 15$, the mass concentration decreases
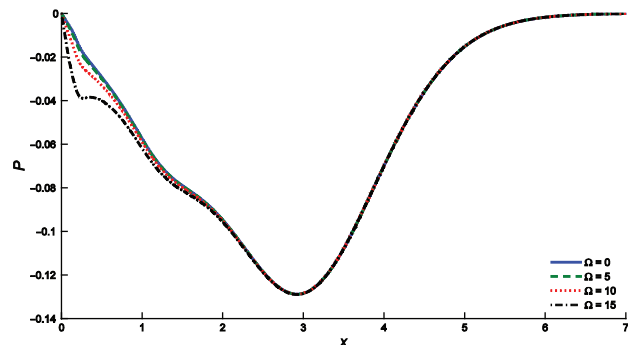

Fig. 7. Chemical potential distribution.
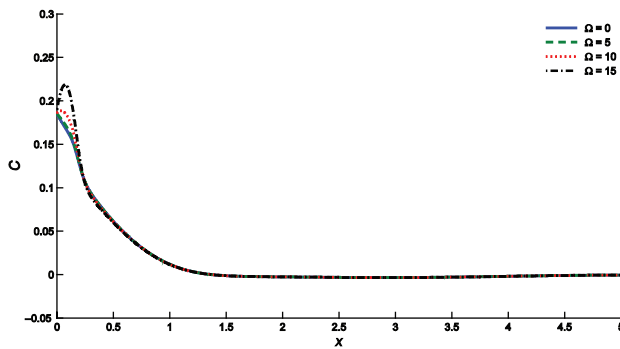
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Fig. 8. Concentration distribution.

with the distance from boundary plane $x = 0$. The effect of rotation on mass concentration C is observed for distances near to the boundary plane $x = 0$ for larger values of rotation parameter Ω.

6. CONCLUSION

The displacement, stress, temperature, chemical potential and mass concentration fields are examined in a homogeneous, isotropic and rotating generalized thermoelastic half-space with diffusion by using finite element method. The rotation parameter has a significant effect on all the fields at distances near to the boundary plane $x = 0$.

NOMENCLATURE

u_i The components of the displacement vector
θ The increment in temperature over the uniform temperature
C The mass concentration
P The chemical potential
T_0 The uniform temperature
e_{ij} The components of strain tensor ($e_{ii} = e$)
σ_{ij} The components of stress tensor
δ_{ij} The Kronecker delta
ρ The density of the medium
c_e The specific heat at constant strain
K The coefficient of thermal conductivity
τ_0 The thermal relaxation time
τ_1 The diffusion relaxation time
Ω The rotation vector
D The thermodiffusion constant
a The measure of thermodiffusion effects
b The measure of diffusion effects
λ, μ The Lame's constants
α_t The coefficient of linear thermal expansion
α_c The coefficient of linear diffusion expansion

$$\beta = (3\lambda + 2\mu)\alpha_c$$
$$\gamma = (3\lambda + 2\mu)\alpha_t.$$

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